7-5 Parametric Equations

Sketch the curve given by each pair of parametric equations over the given interval.

1. \( x = t^2 + 3 \) and \( y = \frac{t}{4} - 5; -5 \leq t \leq 5 \)

**SOLUTION:**

Make a table of values for \(-5 \leq t \leq 5\).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>28</td>
<td>-6.25</td>
</tr>
<tr>
<td>-4</td>
<td>19</td>
<td>-6</td>
</tr>
<tr>
<td>-3</td>
<td>12</td>
<td>-5.75</td>
</tr>
<tr>
<td>-2</td>
<td>7</td>
<td>-5.5</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
<td>-5.25</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>-4.75</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>-4.5</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>-4.25</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
<td>-4</td>
</tr>
<tr>
<td>5</td>
<td>28</td>
<td>-3.75</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \(t\)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \(t\) moves from \(-5\) to \(5\).
7-5 Parametric Equations

2. \( x = \frac{t^2}{2} \) and \( y = -4t; -4 \leq t \leq 4 \)

\textit{SOLUTION:}

Make a table of values for \(-4 \leq t \leq 4\).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>-3</td>
<td>4.5</td>
<td>12</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>-1</td>
<td>0.5</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>-8</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
<td>-12</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>-16</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \(t\)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \(t\) moves from \(-4\) to 4.
3. \( x = -\frac{5t}{2} + 4 \) and \( y = t^2 - 8; -6 \leq t \leq 6 \)

**SOLUTION:**

Make a table of values for \(-6 \leq t \leq 6\).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>19</td>
<td>28</td>
</tr>
<tr>
<td>-5</td>
<td>16.5</td>
<td>17</td>
</tr>
<tr>
<td>-4</td>
<td>14</td>
<td>8</td>
</tr>
<tr>
<td>-3</td>
<td>11.5</td>
<td>1</td>
</tr>
<tr>
<td>-2</td>
<td>9</td>
<td>-4</td>
</tr>
<tr>
<td>-1</td>
<td>6.5</td>
<td>-7</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>-8</td>
</tr>
<tr>
<td>1</td>
<td>1.5</td>
<td>-7</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>-4</td>
</tr>
<tr>
<td>3</td>
<td>-3.5</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>-6</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>-8.5</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>-11</td>
<td>28</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \(t\)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \(t\) moves from \(-6\) to \(6\).
4. \( x = 3t + 6 \) and \( y = \sqrt{t} + 1; \ 0 \leq t \leq 9 \\)

\textit{SOLUTION:} \\
Make a table of values for \( 0 \leq t \leq 9. \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>2.4</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>2.7</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
<td>3.2</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
<td>3.4</td>
</tr>
<tr>
<td>7</td>
<td>27</td>
<td>3.6</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
<td>3.8</td>
</tr>
<tr>
<td>9</td>
<td>33</td>
<td>4</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( t\)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( t \) moves from 0 to 9.
5. \( x = 2t - 1 \) and \( y = -\frac{t^2}{2} + 7; -4 \leq t \leq 4 

**SOLUTION:**

Make a table of values for \(-4 \leq t \leq 4\).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-9</td>
<td>-1</td>
</tr>
<tr>
<td>-3</td>
<td>-7</td>
<td>2.5</td>
</tr>
<tr>
<td>-2</td>
<td>-5</td>
<td>5</td>
</tr>
<tr>
<td>-1</td>
<td>-3</td>
<td>6.5</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>6.5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>-1</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \(t\)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( t \) moves from \(-4 \) to \( 4 \).
7-5 Parametric Equations

6. \( x = -2t^2 \) and \( y = \frac{t}{3} - 6; -6 \leq t \leq 6 \)

**SOLUTION:**

Make a table of values for \(-6 \leq t \leq 6\).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>-72</td>
<td>-8</td>
</tr>
<tr>
<td>-5</td>
<td>-50</td>
<td>-7.7</td>
</tr>
<tr>
<td>-4</td>
<td>-32</td>
<td>-7.3</td>
</tr>
<tr>
<td>-3</td>
<td>-18</td>
<td>-7</td>
</tr>
<tr>
<td>-2</td>
<td>-8</td>
<td>-6.7</td>
</tr>
<tr>
<td>-1</td>
<td>-2</td>
<td>-6.3</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>-6</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
<td>-5.7</td>
</tr>
<tr>
<td>2</td>
<td>-8</td>
<td>-5.3</td>
</tr>
<tr>
<td>3</td>
<td>-18</td>
<td>-5</td>
</tr>
<tr>
<td>4</td>
<td>-32</td>
<td>-4.7</td>
</tr>
<tr>
<td>5</td>
<td>-50</td>
<td>-4.3</td>
</tr>
<tr>
<td>6</td>
<td>-72</td>
<td>-4</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( t \)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( t \) moves from \(-6\) to \(6\).
7-5 Parametric Equations

7. \( x = \frac{t}{2} \) and \( y = -\sqrt{t} + 5; \ 0 \leq t \leq 8 \)

**SOLUTION:**

Make a table of values for \( 0 \leq t \leq 8 \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3.6</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>3.3</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>2.5</td>
<td>2.8</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>2.6</td>
</tr>
<tr>
<td>7</td>
<td>3.5</td>
<td>2.4</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( t \)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( t \) moves from 0 to 8.
8. \( x = t^2 - 4 \) and \( y = 3t - 8; \ -5 \leq t \leq 5 \)

**SOLUTION:**
Make a table of values for \(-5 \leq t \leq 5\).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>21</td>
<td>-23</td>
</tr>
<tr>
<td>-4</td>
<td>12</td>
<td>-20</td>
</tr>
<tr>
<td>-3</td>
<td>5</td>
<td>-17</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
<td>-14</td>
</tr>
<tr>
<td>-1</td>
<td>-3</td>
<td>-11</td>
</tr>
<tr>
<td>0</td>
<td>-4</td>
<td>-8</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
<td>-5</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
<td>7</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \(t\)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \(t\) moves from \(-5\) to \(5\).
7-5 Parametric Equations

Write each pair of parametric equations in rectangular form. Then graph the equation and state any restrictions on the domain.

9. \( x = 2t - 5, y = t^2 + 4 \)

**SOLUTION:**
Solve for \( t \) in the parametric equation for \( x \).

\[
\begin{align*}
x &= 2t - 5 \\
x + 5 &= 2t \\
\frac{x + 5}{2} &= t
\end{align*}
\]

Substitute for \( t \) in the parametric equation for \( y \).

\[
\begin{align*}
y &= t^2 + 4 \\
&= \left(\frac{x + 5}{2}\right)^2 + 4 \\
&= \frac{x^2 + 10x + 25}{4} + 4 \\
&= \frac{x^2}{4} + \frac{10x}{4} + \frac{25}{4} + 4 \\
&= 0.25x^2 + 2.5x + 10.25
\end{align*}
\]

Make a table of values to graph \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-13</td>
<td>20</td>
<td>-3</td>
<td>5</td>
</tr>
<tr>
<td>-11</td>
<td>13</td>
<td>-1</td>
<td>8</td>
</tr>
<tr>
<td>-9</td>
<td>8</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>-7</td>
<td>5</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>-5</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( t \) increases. Since substituting larger values for \( t \) in \( x = 2t - 5 \) produces increasing values of \( x \), the orientation moves from left to right.
7-5 Parametric Equations

10. \(x = 3t + 9, \ y = t^2 - 7\)

**SOLUTION:**

Solve for \(t\) in the parametric equation for \(x\).

\[
x = 3t + 9
\]
\[
x - 9 = 3t
\]
\[
\frac{x - 9}{3} = t
\]

Substitute for \(t\) in the parametric equation for \(y\).

\[
y = t^2 - 7
\]
\[
= \left(\frac{x - 9}{3}\right)^2 - 7
\]
\[
= \frac{x^2 - 18x + 81}{9} - 7
\]
\[
= \frac{x^2}{9} - \frac{18x}{9} + \frac{81}{9} - 7
\]
\[
= \frac{x^2}{9} - 2x + 2
\]

Make a table of values to graph \(y\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>9</td>
<td>9</td>
<td>-7</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>12</td>
<td>-6</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
<td>15</td>
<td>-3</td>
</tr>
<tr>
<td>6</td>
<td>-6</td>
<td>18</td>
<td>2</td>
</tr>
<tr>
<td>-5</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \(t\) increases. Since substituting larger values for \(t\) in \(x = 3t + 9\) produces increasing values of \(x\), the orientation moves from left to right.
7-5 Parametric Equations

11. \( x = t^2 - 2, \ y = 5t \)

**SOLUTION:**

Solve for \( t \) in the parametric equation for \( y \).

\[ y = 5t \]

\[ \frac{y}{5} = t \]

Substitute for \( t \) in the parametric equation for \( x \).

\[ x = t^2 - 2 \]

\[ x = \left(\frac{y}{5}\right)^2 - 2 \]

\[ x + 2 = \left(\frac{y}{5}\right)^2 \]

\[ \pm \sqrt{x + 2} = \frac{y}{5} \]

\[ \pm 5\sqrt{x + 2} = y \]

Make a table of values to graph \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>-5, 5</td>
</tr>
<tr>
<td>0</td>
<td>-7.1, 7.1</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( t \) increases. Since substituting larger values for \( t \) in \( y = 5t \) produces increasing values of \( y \), the orientation moves from bottom to top.
12. \( x = t^2 + 1, y = -4t + 3 \)

\[ \text{SOLUTION:} \]

Solve for \( t \) in the parametric equation for \( y \).
\[
y = -4t + 3
\]
\[
y - 3 = -4t
\]
\[
\frac{y - 3}{-4} = t
\]

Substitute for \( t \) in the parametric equation for \( x \).
\[
x = t^2 + 1
\]
\[
x = \left( \frac{y - 3}{-4} \right)^2 + 1
\]
\[
x - 1 = \left( \frac{y - 3}{-4} \right)^2
\]
\[
\pm \sqrt{x - 1} = \frac{y - 3}{-4}
\]

\(-4 \pm \sqrt{x - 1} + 3 = y\)

Make a table of values to graph \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>-7, 7</td>
</tr>
<tr>
<td>5</td>
<td>-11, 11</td>
</tr>
<tr>
<td>10</td>
<td>-15, 15</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( t \) increases. Since substituting larger values for \( t \) in \( y = -4t + 3 \) produces decreasing values of \( y \), the orientation moves from top to bottom.
7-5 Parametric Equations

13. \( x = -t - 4, y = 3t^2 \)

_SOLUTION:

Solve for \( t \) in the parametric equation for \( x \).
\[
\begin{align*}
    x &= -t - 4 \\
    x + 4 &= -t \\
    -x - 4 &= t
\end{align*}
\]

Substitute for \( t \) in the parametric equation for \( y \).
\[
\begin{align*}
    y &= 3t^2 \\
    y &= 3(-x - 4)^2 \\
    y &= 3(x^2 + 8x + 16) \\
    y &= 3x^2 + 24x + 48
\end{align*}
\]

Make a table of values to graph \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>12</td>
</tr>
<tr>
<td>-5</td>
<td>3</td>
</tr>
<tr>
<td>-4</td>
<td>0</td>
</tr>
<tr>
<td>-3</td>
<td>3</td>
</tr>
<tr>
<td>-2</td>
<td>12</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( t \) increases. Since substituting larger values for \( t \) in \( x = -t - 4 \) produces decreasing values of \( x \), the orientation moves from right to left.
14. \( x = 5t - 1, \ y = 2t^2 + 8 \)

**SOLUTION:**

Solve for \( t \) in the parametric equation for \( x \).
\[
\begin{align*}
  x &= 5t - 1 \\
  x + 1 &= 5t \\
  \frac{x + 1}{5} &= t 
\end{align*}
\]

Substitute for \( t \) in the parametric equation for \( y \).
\[
\begin{align*}
  y &= 2t^2 + 8 \\
  y &= 2\left(\frac{x + 1}{5}\right)^2 + 8 \\
  y &= 2\left(\frac{x^2 + 2x + 1}{25}\right) + 8 \\
  y &= \frac{2x^2 + 4x + 2}{25} + 8 \\
  y &= \frac{2x^2}{25} + \frac{4x}{25} + \frac{202}{25} 
\end{align*}
\]

Make a table of values to graph \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-16</td>
<td>26</td>
</tr>
<tr>
<td>-11</td>
<td>16</td>
</tr>
<tr>
<td>-6</td>
<td>10</td>
</tr>
<tr>
<td>-1</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>16</td>
</tr>
<tr>
<td>14</td>
<td>26</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( t \) increases. Since substituting larger values for \( t \) in \( x = 5t - 1 \) produces increasing values of \( x \), the orientation moves from left to right.

15. \( x = 4t^2, \ y = \frac{6t}{5} + 9 \)

**SOLUTION:**
7-5 Parametric Equations

Solve for \( t \) in the parametric equation for \( y \).

\[
\begin{align*}
y &= \frac{6t}{5} + 9 \\
y - 9 &= \frac{6t}{5} \\
\frac{5(y - 9)}{6} &= t
\end{align*}
\]

Substitute for \( t \) in the parametric equation for \( x \).

\[
\begin{align*}
x &= 4t^2 \\
x &= 4 \left( \frac{5(y - 9)}{6} \right)^2 \\
x &= \frac{4}{4} \left( \frac{5(y - 9)}{6} \right)^2 \\
x &= \frac{5(y - 9)}{6} \\
\pm \frac{5\sqrt{x}}{6} &= y - 9 \\
\pm \frac{3\sqrt{x}}{5} &= y - 9 \\
\pm \frac{3\sqrt{x}}{5} + 9 &= y
\end{align*}
\]

Make a table of values to graph \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>7.8, 10.2</td>
</tr>
<tr>
<td>8</td>
<td>7.3, 10.7</td>
</tr>
<tr>
<td>12</td>
<td>6.9, 11.1</td>
</tr>
<tr>
<td>16</td>
<td>6.6, 11.4</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( t \) increases. Since substituting larger values for \( t \) in \( y = \frac{6t}{5} + 9 \) produces increasing values of \( y \), the orientation moves from bottom to top.
7-5 Parametric Equations

16. \( x = \frac{t}{3} + 2, \ y = \frac{t^2}{6} - 7 \)

\[ \text{SOLUTION:} \]

Solve for \( t \).

\[
x = \frac{t}{3} + 2
\]

\[
x - 2 = \frac{t}{3}
\]

\[
3x - 6 = t
\]

Substitute for \( t \).

\[
y = \frac{t^2}{6} - 7
\]

\[
= \frac{(3x - 6)^2}{6} - 7
\]

\[
= \frac{9x^2 - 36x + 36}{6} - 7
\]

\[
= 1.5x^2 - 6x + 6 - 7
\]

\[
= 1.5x^2 - 6x - 1
\]

Make a table of values to graph \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>17</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-7</td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( t \) increases. Since substituting larger values for \( t \) in \( x = \frac{t}{3} + 2 \) produces increasing values of \( x \), the orientation moves from left to right.
7-5 Parametric Equations

17. MOVIE STUNTS  During the filming of a movie, a stunt double leaps off the side of a building. The pulley system connected to the stunt double allows for a vertical fall modeled by \( y = -16t^2 + 15t + 100 \), and a horizontal movement modeled by \( x = 4t \), where \( x \) and \( y \) are measured in feet and \( t \) is measured in seconds. Write and graph an equation in rectangular form to model the stunt double’s fall for \( 0 \leq t \leq 3 \).

**SOLUTION:**
Solve for \( t \).
\[
x = 4t \\
\frac{x}{4} = t
\]
Substitute for \( t \).
\[
y = -16t^2 + 15t + 100 \\
y = -16 \left( \frac{x}{4} \right)^2 + 15 \left( \frac{x}{4} \right) + 100 \\
y = -16 \frac{x^2}{16} + 15 \frac{x}{4} + 100 \\
y = -x^2 + \frac{15x}{4} + 100
\]
Make a table of values for \( 0 \leq t \leq 3 \).
<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>99</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>66</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>1</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( t\)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( t \) moves from 0 to 3.
7-5 Parametric Equations

Write each pair of parametric equations in rectangular form. Then graph the equation.

18. \( x = 3 \cos \theta \) and \( y = 5 \sin \theta \)

\textbf{SOLUTION:}
Solve the equations for \( \cos \theta \) and \( \sin \theta \). Then use a trigonometric identity.

\[
\begin{align*}
x &= 3 \cos \theta \\
\frac{x}{3} &= \cos \theta \\
y &= 5 \sin \theta \\
\frac{y}{5} &= \sin \theta \\
\sin^2 \theta + \cos^2 \theta &= 1 \\
\left(\frac{y}{5}\right)^2 + \left(\frac{x}{3}\right)^2 &= 1 \\
\frac{y^2}{25} + \frac{x^2}{9} &= 1
\end{align*}
\]

The equation is the general form of an ellipse that has a center at the origin, a vertical major axis of length 10, and a horizontal minor axis of length 6.

Make a table of values for \( 0 \leq \theta \leq 2\pi \).

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>( \pi )</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{3\pi}{2} )</td>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>( 2\pi )</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( \theta \)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( \theta \) moves from 0 to \( 2\pi \).
7-5 Parametric Equations

19. \( x = 7 \sin \theta \) and \( y = 2 \cos \theta \)

**SOLUTION:**
Solve the equations for \( \sin \theta \) and \( \cos \theta \). Then use a trigonometric identity.

\[
\begin{align*}
\frac{x}{7} & = \sin \theta \\
y & = 2 \cos \theta \\
\frac{y}{2} & = \cos \theta
\end{align*}
\]

\[
\begin{align*}
\sin^2 \theta + \cos^2 \theta & = 1 \\
\left( \frac{x}{7} \right)^2 + \left( \frac{y}{2} \right)^2 & = 1 \\
\frac{x^2}{49} + \frac{y^2}{4} & = 1
\end{align*}
\]

The equation is the general form of an ellipse that has a center at the origin, a horizontal major axis of length 14, and a vertical minor axis of length 4.

Make a table of values for \( 0 \leq \theta \leq 2\pi \).

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>( \pi )</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>( \frac{3\pi}{2} )</td>
<td>-7</td>
<td>0</td>
</tr>
<tr>
<td>( 2\pi )</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( \theta \)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( \theta \) moves from 0 to \( 2\pi \).
7-5 Parametric Equations

20. \( x = 6 \cos \theta \) and \( y = 4 \sin \theta \)

**SOLUTION:**
Solve the equations for \( \cos \theta \) and \( \sin \theta \). Then use a trigonometric identity.

\[
\begin{align*}
x &= 6 \cos \theta \\
\frac{x}{6} &= \cos \theta \\
y &= 4 \sin \theta \\
\frac{y}{4} &= \sin \theta \\
\sin^2 \theta + \cos^2 \theta &= 1 \\
\left(\frac{y}{4}\right)^2 + \left(\frac{x}{6}\right)^2 &= 1 \\
\frac{y^2}{16} + \frac{x^2}{36} &= 1
\end{align*}
\]

The equation is the general form of an ellipse that has a center at the origin, a horizontal major axis of length 12, and a vertical minor axis of length 8.

Make a table of values for \( 0 \leq \theta \leq 2\pi \).

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>( \pi )</td>
<td>-6</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{3\pi}{2} )</td>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>( 2\pi )</td>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

Plot the \( (x, y) \) coordinates for each \( \theta \)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( \theta \) moves from 0 to \( 2\pi \).
7-5 Parametric Equations

21. \(x = 3 \cos \theta \) and \(y = 3 \sin \theta\)

**SOLUTION:**
Solve the equations for \(\cos \theta\) and \(\sin \theta\). Then use a trigonometric identity.

\[
\frac{x}{3} = \cos \theta
\]
\[
\frac{y}{3} = \sin \theta
\]

\[
\sin^2 \theta + \cos^2 \theta = 1
\]
\[
\left(\frac{y}{3}\right)^2 + \left(\frac{x}{3}\right)^2 = 1
\]
\[
\frac{y^2}{9} + \frac{x^2}{9} = 1
\]

The equation can be written in the general form of a circle, \(x^2 + y^2 = 9\), that has a center at the origin and radius 3.

Make a table of values for \(0 \leq \theta \leq 2\pi\).

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>(\frac{\pi}{2})</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>(\pi)</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>(\frac{3\pi}{2})</td>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>(2\pi)</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \(\theta\)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \(\theta\) moves from 0 to \(2\pi\).
22. \( x = 8 \sin \theta \) and \( y = \cos \theta \)

**SOLUTION:**

Solve the equations for \( \sin \theta \) and \( \cos \theta \). Then use a trigonometric identity.

\[
\frac{x}{8} = \sin \theta
\]

\[
y = \cos \theta
\]

\[
\cos \theta = y
\]

\[
\sin^2 \theta + \cos^2 \theta = 1
\]

\[
\left( \frac{x}{8} \right)^2 + y^2 = 1
\]

\[
\frac{x^2}{64} + y^2 = 1
\]

The equation is the general form of an ellipse that has a center at the origin, a horizontal major axis of length 16 and a vertical minor axis of length 2.

Make a table of values for \( 0 \leq \theta \leq 2\pi \).

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>( \pi )</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>( \frac{3\pi}{2} )</td>
<td>-8</td>
<td>0</td>
</tr>
<tr>
<td>2( \pi )</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Plot the \( (x, y) \) coordinates for each \( \theta \)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( \theta \) moves from 0 to \( 2\pi \).
7-5 Parametric Equations

23. \( x = 5 \cos \theta \) and \( y = 6 \sin \theta \)

**SOLUTION:**

Solve the equations for \( \cos \theta \) and \( \sin \theta \). Then use a trigonometric identity.

\[
\begin{align*}
\frac{x}{5} &= \cos \theta \\
\frac{y}{6} &= \sin \theta
\end{align*}
\]

\[
\sin^2 \theta + \cos^2 \theta = 1
\]

\[
\left(\frac{y}{6}\right)^2 + \left(\frac{x}{5}\right)^2 = 1
\]

\[
\frac{y^2}{36} + \frac{x^2}{25} = 1
\]

The equation is the general form of an ellipse that has a center at the origin, a vertical major axis of length 12, and a horizontal minor axis of length 10.

Make a table of values for \( 0 \leq \theta \leq 2\pi \).

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>( \pi )</td>
<td>-5</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{3\pi}{2} )</td>
<td>0</td>
<td>-6</td>
</tr>
<tr>
<td>( 2\pi )</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( \theta \)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( \theta \) moves from 0 to \( 2\pi \).
24. \( x = 10 \sin \theta \) and \( y = 9 \cos \theta \)

**SOLUTION:**

Solve the equations for \( \cos \theta \) and \( \sin \theta \). Then use a trigonometric identity.

\[
\frac{y}{9} = \cos \theta
\]

\[
x = 10 \sin \theta
\]

\[
\frac{x}{10} = \sin \theta
\]

\[
\sin^2 \theta + \cos^2 \theta = 1
\]

\[
\left( \frac{x}{10} \right)^2 + \left( \frac{y}{9} \right)^2 = 1
\]

\[
\frac{x^2}{100} + \frac{y^2}{81} = 1
\]

The equation is the general form of an ellipse that has a center at the origin, a horizontal major axis of length 20 and a vertical minor axis of length 18.

Make a table of values for \( 0 \leq \theta \leq 2\pi \).

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>( \pi )</td>
<td>0</td>
<td>-9</td>
</tr>
<tr>
<td>( \frac{3\pi}{2} )</td>
<td>-10</td>
<td>0</td>
</tr>
<tr>
<td>( 2\pi )</td>
<td>0</td>
<td>9</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( \theta \)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( \theta \) moves from 0 to 2\( \pi \).
7-5 Parametric Equations

25. \( x = \sin \theta \) and \( y = 7 \cos \theta \)

**SOLUTION:**

Solve the equations for \( \cos \theta \) and \( \sin \theta \). Then use a trigonometric identity
\[
x = \sin \theta \\
\sin \theta = x
\]

\[
y = 7 \cos \theta \\
\frac{y}{7} = \cos \theta
\]

\[
\sin^2 \theta + \cos^2 \theta = 1 \\
x^2 + \left(\frac{y}{7}\right)^2 = 1 \\
x^2 + \frac{y^2}{49} = 1
\]

The equation is the general form of an ellipse that has a center at the origin, a vertical major axis of length 14 and a horizontal minor axis of length 2.

Make a table of values for \( 0 \leq \theta \leq 2\pi \).

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( \pi )</td>
<td>0</td>
<td>-7</td>
</tr>
<tr>
<td>( \frac{3\pi}{2} )</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>2\pi</td>
<td>0</td>
<td>7</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( \theta \)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve as \( \theta \) moves from 0 to \( 2\pi \).
7-5 Parametric Equations

Use each parameter to write the parametric equations that can represent each equation. Then graph the equations, indicating the speed and orientation.

26. \( t = 3x - 2; y = x^2 + 9 \)

**SOLUTION:**

\[
\begin{align*}
    t &= 3x - 2 \\
    t + 2 &= 3x \\
    \frac{t + 2}{3} &= x \\
    y &= x^2 + 9 \\
    y &= \left(\frac{t + 2}{3}\right)^2 + 9 \\
    y &= \frac{t^2 + 4t + 4}{9} + 9 \\
    y &= \frac{t^2 + 4t + 4 + 81}{9} \\
    y &= \frac{t^2 + 4t + 85}{9}
\end{align*}
\]

Make a table of values for \( t \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-11</td>
<td>-3</td>
<td>18</td>
</tr>
<tr>
<td>-8</td>
<td>-2</td>
<td>13</td>
</tr>
<tr>
<td>-5</td>
<td>-1</td>
<td>10</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>18</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( t \)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve.
7-5 Parametric Equations

27. \( t = 8x; \quad y^2 = 9 - x^2 \)

**SOLUTION:**

\[
\begin{align*}
\frac{t}{8} &= x \\
y^2 &= 9 - x^2 \\
y^2 &= 9 - \left( \frac{t}{8} \right)^2 \\
y^2 &= 9 - \frac{t^2}{64} \\
y &= \sqrt{9 - \frac{t^2}{64}}
\end{align*}
\]

Make a table of values for \( t \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-24</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>24</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( t\)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve.
7-5 Parametric Equations

28. \( t = 2 - \frac{x}{3}; y = \frac{x^2}{12} \)

**SOLUTION:**

\[
\begin{align*}
t & = 2 - \frac{x}{3} \\
t - 2 & = -\frac{x}{3} \\
-3(t - 2) & = x \\
6 - 3t & = x \\
y & = \frac{x^2}{12} \\
y & = \frac{(6 - 3t)^2}{12} \\
y & = 36 - 36t + 9t^2 \\
y & = \frac{9t^2 - 36t + 36}{12} \\
y & = \frac{3t^2 - 3t + 3}{4}
\end{align*}
\]

Make a table of values for \( t \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>18</td>
<td>27</td>
</tr>
<tr>
<td>-2</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-6</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>-12</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>-18</td>
<td>27</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( t\)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve.
7-5 Parametric Equations

29. \( t = \frac{x}{5} + 4; \ y = 10 - x^2 \)

**SOLUTION:**

\[
\begin{align*}
t &= \frac{x}{5} + 4 \\
t - 4 &= \frac{x}{5} \\
5(t - 4) &= x \\
5t - 20 &= x \\
\end{align*}
\]

\[
\begin{align*}
y &= 10 - x^2 \\
y &= 10 - (5t - 20)^2 \\
y &= 10 - (25t^2 - 200t + 400) \\
y &= -25t^2 + 200t - 390 \\
\end{align*}
\]

Make a table of values for \( t \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-10</td>
<td>-90</td>
</tr>
<tr>
<td>3</td>
<td>-5</td>
<td>-15</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>-15</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>-90</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( t \)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve.

30. \( t = 4x + 7; \ y = \frac{x^2 - 1}{2} \)

**SOLUTION:**

\[
\begin{align*}
t &= 4x + 7 \\
t - 7 &= 4x \\
\frac{t - 7}{4} &= x \\
\end{align*}
\]

\[
\begin{align*}
y &= \frac{x^2 - 1}{2} \\
\end{align*}
\]
7-5 Parametric Equations

\[ y = \left( \frac{t-7}{4} \right)^2 - 1 \]
\[ y = \frac{t^2-14t+49}{16} - 1 \]
\[ y = \frac{t^2-14t+49}{16} \cdot \frac{1}{2} \]
\[ y = \frac{t^2-14t+33}{16} \]
\[ y = \frac{t^2-14t+33}{32} \]

Make a table of values for \( t \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-21</td>
<td>-7</td>
<td>24</td>
</tr>
<tr>
<td>-13</td>
<td>-5</td>
<td>12</td>
</tr>
<tr>
<td>-5</td>
<td>-3</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>19</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>27</td>
<td>5</td>
<td>12</td>
</tr>
<tr>
<td>35</td>
<td>7</td>
<td>24</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( t\)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve.
**7-5 Parametric Equations**

31. \( t = \frac{1-x}{2}; y = \frac{3-x^2}{4} \)

**SOLUTION:**

\[
\begin{align*}
  t &= \frac{1-x}{2} \\
  2t &= 1-x \\
  1-2t &= x \\

  y &= \frac{3-x^2}{4} \\
  y &= \frac{3-(1-2t)^2}{4} \\
  y &= \frac{3-(1-4t+4t^2)}{4} \\
  y &= \frac{-4t^2+4t+2}{4} \\
  y &= -t^2+t+\frac{1}{2} \\

\end{align*}
\]

Make a table of values for \( t \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>7</td>
<td>-11.5</td>
</tr>
<tr>
<td>-2</td>
<td>5</td>
<td>-5.5</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
<td>-1.5</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
<td>-1.5</td>
</tr>
<tr>
<td>3</td>
<td>-5</td>
<td>5.5</td>
</tr>
<tr>
<td>4</td>
<td>-7</td>
<td>-11.5</td>
</tr>
</tbody>
</table>

Plot the \((x, y)\) coordinates for each \( t\)-value and connect the points to form a smooth curve. The arrows in the graph indicate the orientation of the curve.
32. **BASEBALL** A baseball player hits the ball at a 28° angle with an initial speed of 103 feet per second. The bat is 4 feet from the ground at the time of impact. Assuming that the ball is not caught, determine the distance traveled by the ball.

![Baseball Image]

**SOLUTION:**
To determine the distance the ball travels, you need the horizontal distance that the ball has traveled when the ball hits the ground or when the height of the ball is 0. First, write a parametric equation for the vertical position of the ball.

\[ y = t v_0 \sin \theta - \frac{1}{2} g t^2 + h_0 \]

\[ = t(103) \sin 28 - \frac{1}{2} (32)t^2 + 4 \]

Graph the equation for the vertical position. Use 2: **zero** function on the **CALC** menu to find the time \( t \) for when the ball will hit the ground. The value is about 3.1 seconds.

![Graph Image]

Determine the horizontal position of the ball at 3.1 seconds.

\[ x = t v_0 \cos \theta \]

\[ = 3.1(103) \cos 28 \]

\[ \approx 281.9 \]

The ball will travel a distance of about 282 feet.
33. **FOOTBALL** Delmar attempts a 43–yard field goal. He kicks the ball at a 41° angle with an initial speed of 70 feet per second. The goal post is 15 feet high. Is the kick long enough to make the field goal?

**SOLUTION:**

To determine whether Delmar makes the field goal, you need the horizontal distance that the ball has traveled when the height of the ball is 15 feet. First, write a parametric equation for the vertical position of the ball.

\[
y = t v_0 \sin \theta - \frac{1}{2} g t^2 + h_0
\]

\[
= t (70) \sin 41 - \frac{1}{2} (32) t^2 + 0
\]

Graph the equation for the vertical position and the line \( y = 15 \). The curve will intersect the line in two places. The second intersection represents the ball as it is moving down toward goal post. Use **5: intersect** function on the **CALC** menu to find the second point of intersection with \( y = 15 \). The value is about 2.49 seconds.

Determine the horizontal position of the ball at 2.49 seconds.

\[
x = t v_0 \cos \theta
\]

\[
= 2.49 (70) \cos 41
\]

\[
\approx 131.55
\]

The ball will travel a distance of about 132 feet before the height reaches 15 feet a second time. There are 3 feet in a yard, so the distance of the field goal is 43 \( \cdot \) 3 or 129 feet. Since the vertical position of the ball will not reach 15 feet until it achieves a horizontal distance of 132 feet, Delmar will make the field goal.
7-5 Parametric Equations

Write each pair of parametric equations in rectangular form. Then state the restriction on the domain.

34. \( x = \sqrt{t} + 4 \\
\quad y = 4t + 3 \)

**SOLUTION:**

Solve for \( t \).

\[
x = \sqrt{t} + 4 \\
x - 4 = \sqrt{t} \\
(x - 4)^2 = t
\]

Substitute.

\[
y = 4t + 3 \\
y = 4(x - 4)^2 + 3 \\
y = 4(x^2 - 8x + 16) + 3 \\
y = 4x^2 - 32x + 67
\]

From the parametric equation \( x = \sqrt{t} + 4 \), the only possible values for \( x \) are values greater than or equal to four. The domain of the rectangular equation needs to be restricted to \( x \geq 4 \).

35. \( x = \log t \\
\quad y = t + 3 \)

**SOLUTION:**

Solve for \( t \).

\[
x = \log t \\
10^x = t
\]

Substitute.

\[
y = t + 3 \\
y = 10^x + 3
\]

From the parametric equation \( x = \log t \), \( x \) can be all real numbers. Therefore, there is no restriction on the domain.

36. \( x = \sqrt{t - 7} \\
\quad y = -3t - 8 \)

**SOLUTION:**

Solve for \( t \).

\[
x = \sqrt{t - 7} \\
x^2 = t - 7 \\
x^2 + 7 = t
\]

Substitute.

\[
y = -3t - 8 \\
y = -3(x^2 + 7) - 8 \\
y = -3x^2 - 21 - 8 \\
y = -3x^2 - 29
\]

From the parametric equation \( x = \sqrt{t - 7} \), the only possible values for \( x \) are values greater than or equal to zero. The domain of the rectangular equation needs to be restricted to \( x \geq 0 \).
7-5 Parametric Equations

37. \( x = \log (t - 4) \)
\( y = t \)

**SOLUTION:**
Solve for \( t \).
\[ \begin{align*}
  x &= \log (t - 4) \\
  10^x &= t - 4 \\
  10^x + 4 &= t 
\end{align*} \]
Substitute.
\[ \begin{align*}
  y &= t \\
  y &= 10^x + 4 \\
  \text{From the parametric equation } x = \log (t - 4), x \text{ can be all real numbers. Therefore, there is no restriction on the domain.}
\]

38. \( x = \frac{1}{\sqrt{t + 3}} \)
\( y = t \)

**SOLUTION:**
Solve for \( t \).
\[ \begin{align*}
  x &= \frac{1}{\sqrt{t + 3}} \\
  \frac{1}{x} &= \sqrt{t + 3} \\
  \frac{1}{x^2} &= t + 3 \\
  \frac{1}{x^2} - 3 &= t 
\end{align*} \]
Substitute.
\[ \begin{align*}
  y &= t \\
  y &= \frac{1}{x^2} - 3 \\
  \text{From the parametric equation } x = \frac{1}{\sqrt{t + 3}}, \text{ the only possible values for } x \text{ are values greater than zero. The domain of the rectangular equation needs to be restricted to } x > 0. \]
7-5 Parametric Equations

39. \( x = \frac{1}{\log(t + 2)} \)
    \( y = 2t - 4 \)

**SOLUTION:**
Solve for \( t \).
\[
\begin{align*}
x &= \frac{1}{\log(t + 2)} \\
\frac{1}{x} &= \log(t + 2) \\
10^{\frac{1}{x}} &= t + 2 \\
10^{\frac{1}{x}} - 2 &= t
\end{align*}
\]
Substitute.
\[
\begin{align*}
y &= 2t - 4 \\
y &= 2\left(10^{\frac{1}{x}} - 2\right) - 4 \\
y &= 2 \cdot 10^{\frac{1}{x}} - 4 - 4 \\
y &= 2 \cdot 10^{\frac{1}{x}} - 8
\end{align*}
\]
From the parametric equation \( x = \frac{1}{\log(t + 2)} \), \( x \) can be all real numbers except 0. The domain of the rectangular equation needs to be restricted to \( x \neq 0 \).

40. **TENNIS** Jill hits a tennis ball 55 centimeters above the ground at an angle of 15° with the horizontal. The ball has an initial speed of 18 meters per second.
    a. Use a graphing calculator to graph the path of the tennis ball using parametric equations.
    b. How long does the ball stay in the air before hitting the ground?
    c. If Jill is 10 meters from the net and the net is 1.5 meters above the ground, will the tennis ball clear the net? If so, by how many meters? If not, by how many meters is the ball short?

**SOLUTION:**
    a. First, write a parametric equation for the vertical position of the ball. Since Jill hits the ball 55 centimeters above the ground, \( h_0 = 0.55 \) meters and \( g = 9.8 \)
    \[
y = tv_0 \sin \theta - \frac{1}{2} gt^2 + h_0 \\
= t(18) \sin 15 - \frac{1}{2}(9.8)t^2 + 0.55
\]
    Write a parametric equation for the horizontal position of the ball.
    \[
x = tv_0 \cos \theta \\
= t(18) \cos 15
\]
    Use a graphing calculator to graph the path using these two parametric equations.
7-5 Parametric Equations

b. Graph the equation for the vertical position. Use 2: zero on the CALC menu to find the time t for when the ball will hit the ground. The value is about 1.06 seconds.

c. To determine whether the ball will clear the net, you need the vertical position of the ball when the ball has traveled 10 meters. Graph the equation for the horizontal position and the line y = 10. The intersection of the curve and the line represents the time the ball will reach a distance of 10 meters. Use 5: intersect on the CALC menu to find the second point of intersection with y = 10. The value is about 0.575 second.

Determine the vertical position of the ball at 0.575 second.

\[
y = t v_0 \sin \theta - \frac{1}{2} gt^2 + h_0
\]

\[
= 0.575(18) \sin 15 - \frac{1}{2} (9.8)(0.575)^2 + 0.55
\]

\[
= 1.61
\]

At about 0.575, the ball reaches the net and is at a height of about 1.6 meters, so it clears the net.
7-5 Parametric Equations

Write a set of parametric equations for the line or line segment with the given characteristics.

41. line with a slope of 3 that passes through (4, 7)

**SOLUTION:**

Make a table of values. Let \( t = 0 \) represent the first point (4, 7) and let \( t \) be in intervals of 1.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>13</td>
</tr>
</tbody>
</table>

Since the slope of the line is given as 3, the \( y \)–values will increase by 3 units each time \( x \) increases by 1 unit. Find an equation for \( x \) in terms of \( t \). Since \( x \) increases by increments of 1, \( m = 1 \).

\[
\begin{align*}
x &= mt + b \\
4 &= (1)(0) + b \\
4 &= b
\end{align*}
\]

An equation for \( x \) is \( x = t + 4 \). Repeat the process to find an equation for \( y \) in terms of \( t \). Since \( y \) increases by increments of 3, \( m = 3 \).

\[
\begin{align*}
y &= mt + b \\
7 &= (3)(0) + b \\
7 &= b
\end{align*}
\]

An equation for \( y \) is \( y = 3t + 7 \).

42. line with a slope of \(-0.5\) that passes through (3, \(-2\))

**SOLUTION:**

Make a table of values. Let \( t = 0 \) represent the first point (3, \(-2\)) and let \( t \) be in intervals of 1.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>(-2)</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>(-2.5)</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>(-3)</td>
</tr>
</tbody>
</table>

Since the slope of the line is given as \(-0.5\), the \( y \)–values will decrease by 0.5 unit each time \( x \) increases by 1 unit. Find an equation for \( x \) in terms of \( t \). Since \( x \) increases by increments of 1, \( m = 1 \).

\[
\begin{align*}
x &= mt + b \\
3 &= (1)(0) + b \\
3 &= b
\end{align*}
\]

An equation for \( x \) is \( x = t + 3 \). Repeat the process to find an equation for \( y \) in terms of \( t \). Since \( y \) decreases by increments of 0.5, \( m = -0.5 \).

\[
\begin{align*}
y &= mt + b \\
-2 &= (-0.5)(0) + b \\
-2 &= b
\end{align*}
\]

An equation for \( y \) is \( y = -0.5t - 2 \).
7-5 Parametric Equations

43. line segment with endpoints (−2, −6) and (2, 10)

**SOLUTION:**
Find the slope of the line segment with endpoints (−2, −6) and (2, 10).

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ m = \frac{10 - (-6)}{2 - (-2)} \]

\[ m = \frac{16}{4} \]

\[ m = 4 \]

Make a table of values. Let \( t = 0 \) represent the first point (−2, −6) and let \( t \) be in intervals of 1.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>−2</td>
<td>−6</td>
</tr>
<tr>
<td>1</td>
<td>−1</td>
<td>−2</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>

Since the slope of the line is 4, the \( y \)-values will increase by 4 units each time \( x \) increases by 1 unit. Find an equation for \( x \) in terms of \( t \).

Since \( x \) increases by increments of 1, \( m = 1 \).

\[ x = mt + b \]

\[ -2 = (1)(0) + b \]

\[ -2 = b \]

An equation for \( x \) is \( x = t - 2 \). Repeat the process to find an equation for \( y \) in terms of \( t \). Since \( y \) increases by increments of 4, \( m = 4 \).

\[ y = mt + b \]

\[ -6 = (4)(0) + b \]

\[ -6 = b \]

An equation for \( y \) is \( y = 4t - 6 \). Since the parametric equations represent a line segment with endpoints (−2, −6) and (2, 10), the interval for \( t \) is \( 0 \leq t \leq 4 \).
7-5 Parametric Equations

44. line segment with endpoints (7, 13) and (13, 11)

**SOLUTION:**
Find the slope of the line segment with endpoints (7, 13) and (13, 11).

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ m = \frac{13 - 7}{11 - 13} \]
\[ m = \frac{2}{-2} \]
\[ m = -1 \]

Make a table of values. Let \( t = 0 \) represent the first point (7, 13) and let \( t \) be in intervals of 1.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>12.67</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>12.33</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>11.67</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>11.33</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td>11</td>
</tr>
</tbody>
</table>

Since the slope of the line is \(-\frac{1}{3}\), the \( y \)-values will decrease by \(-\frac{1}{3}\) unit each time \( x \) increases by 1 unit. Find an equation for \( x \) in terms of \( t \). Since \( x \) increases by increments of 1, \( m = 1 \).

\[ x = mt + b \]
\[ 7 = (1)(0) + b \]
\[ 7 = b \]

An equation for \( x \) is \( x = t + 7 \). Repeat the process to find an equation for \( y \) in terms of \( t \). Since \( y \) decreases by increments of \(-\frac{1}{3}\), \( m = -\frac{1}{3} \).

\[ y = mt + b \]
\[ -6 = \left( -\frac{1}{3} \right)(0) + b \]
\[ -6 = b \]

An equation for \( y \) is \( y = 4t - 6 \). Since the parametric equations represent a line segment with endpoints \((-2, -6)\) and \((2, 10)\), the interval for \( t \) is \( 0 \leq t \leq 4 \).
7-5 Parametric Equations

Match each set of parametric equations with its graph.

45. \( x = \cos 2t, y = \sin 4t \)

**SOLUTION:**
Use a graphing calculator to graph the parametric equations \( x = \cos 2t \) and \( y = \sin 4t \). Graph in radians.

The answer choice that resembles the graph is b.

46. \( x = \cos 3t, y = \sin t \)

**SOLUTION:**
Use a graphing calculator to graph the parametric equations \( x = \cos 3t \) and \( y = \sin t \). Graph in radians.

The answer choice that resembles the graph is d.
7-5 Parametric Equations

47. \( x = \cos t, y = \sin 3t \)

**SOLUTION:**
Use a graphing calculator to graph the parametric equations \( x = \cos t \) and \( y = \sin 3t \). Graph in radians.

The answer choice that resembles the graph is a.

48. \( x = \cos 4t, y = \sin 3t \)

**SOLUTION:**
Use a graphing calculator to graph the parametric equations \( x = \cos 4t \) and \( y = \sin 3t \). Graph in radians.

The answer choice that resembles the graph is c.

49. **BIOLOGY** A frog jumps off the bank of a creek with an initial velocity of 0.75 meter per second at an angle of 45° with the horizontal. The surface of the creek is 0.3 meter below the edge of the bank. Let \( g \) equal 9.8 meters per second squared.

a. Write the parametric equations to describe the position of the frog at time \( t \). Assume that the surface of the water is located at the line \( y = 0 \).

b. If the creek is 0.5 meter wide, will the frog reach the other bank, which is level with the surface of the creek? If not, how far from the other bank will it hit the water?

c. If the frog was able to jump on a lily pad resting on the surface of the creek 0.4 meter away and stayed in the air for 0.38 second, what was the initial speed of the frog?

**SOLUTION:**

a. The position equations are \( x = tv_0 \cos \theta \) and \( y = tv_0 \sin \theta - \frac{1}{2} gt^2 + h_0 \). The initial velocity \( v_0 \) is 0.75 and \( \theta \) is 45°. The surface of the water is at \( y = 0 \), so the value of the initial height \( h_0 \) is 0.3. By substitution, the position equations are \( x = t \cdot 0.75 \cos 45° \) and \( y = t \cdot 0.75 \sin 45° - 4.9t^2 + 0.3 \).
7-5 Parametric Equations

b. Find the value of \( t \) for which \( y = 0 \) in the vertical position equation.

\[
0 = t \cdot 0.75 \sin 45^\circ - 4.9t^2 + 0.3
\]

\[
0 = \frac{3\sqrt{2}}{8} - 4.9t^2 + 0.3.
\]

Graph the equation for the vertical position. Use 2: zero on the CALC menu to find the time \( t \) for when \( y = 0 \). The value is about 0.3074 second.

Substitute this for \( t \) in \( x = t \cdot 0.75 \cos 45^\circ \), the horizontal position equation.

\[
x = 0.3074(0.75) \cdot \frac{\sqrt{2}}{2} \approx 0.16.
\]

This means that when the frog reaches the surface of the water, he is only about 0.16 m from where he jumped. Therefore, he is about 0.5 – 0.16 or 0.34 meters from the other bank.

c. The 0.4 meter distance is a horizontal distance, so substitute the values into the horizontal position equation and solve for \( v_0 \).

\[
x = tv_0 \cos \theta
\]

\[
0.4 = 0.38 \cdot v_0 \cdot \frac{\sqrt{2}}{2}
\]

\[
1.49 = v_0
\]

50. RACE Luna and Ruby are competing in a 100–meter dash. When the starter gun fires, Luna runs 8.0 meters per second after a 0.1 second delay from the point \((0, 2)\) and Ruby runs 8.1 meters per second after a 0.3 second delay from the point \((0, 5)\).

a. Using the \( y \)-axis as the starting line and assuming that the women run parallel to the \( x \)-axis, write parametric equations to describe each runner’s position after \( t \) seconds.

b. Who wins the race? If the women ran 200 meters instead of 100 meters, who would win? Explain your answer.

**SOLUTION:**

a. Draw a sketch of the race.

Luna and Ruby’s vertical position will stay the same since they are both running parallel to the \( x \)-axis. Thus, \( y = 2 \) for Luna and \( y = 5 \) for Ruby. The runners’ horizontal position is represented by the distance that they have traveled after \( t \) seconds from the \( y \)-axis at a rate \( r \).
\section*{7-5 Parametric Equations}

Using the distance formula, \( d = rt \), Luna’s horizontal position can be described as \( x = 8.0t \). Since Luna has a 0.1 second delay, the equation becomes \( x = 8.0(t - 0.1) \). Ruby’s horizontal position can be similarly described as \( x = 8.1(t - 0.3) \).

\textbf{b.} To decide who wins the race, you need to find the value of \( t \) for when each horizontal function is equal to 100. For Luna,

\begin{align*}
&x = 8.0(t - 0.1) \\
&100 = 8.0(t - 0.1) \\
&12.5 = t - 0.1 \\
&12.6 = t
\end{align*}

For Ruby,

\begin{align*}
&x = 8.1(t - 0.3) \\
&100 = 8.1(t - 0.3) \\
&12.35 = t - 0.3 \\
&12.65 = t
\end{align*}

Luna would win the 100–meter dash by about 0.05 second.

To decide who wins a 200–meter dash, repeat the process solving for \( t \) when \( x = 200 \). For Luna,

\begin{align*}
&x = 8.0(t - 0.1) \\
&200 = 8.0(t - 0.1) \\
&25 = t - 0.1 \\
&25.1 = t
\end{align*}

For Ruby,

\begin{align*}
&x = 8.1(t - 0.3) \\
&200 = 8.1(t - 0.3) \\
&24.69 = t - 0.3 \\
&24.99 = t
\end{align*}

Ruby would win the 200 meter race finishing in 25 seconds, while Luna would finish in 25.1 seconds.

51. \textbf{SOCCER} The graph below models the path of a soccer ball kicked by one player and then headed back by another player. The path of the initial kick is shown in blue, and the path of the headed ball is shown in red.

\textbf{a.} If the ball is initially kicked at an angle of 50°, find the initial speed of the ball.

\textbf{b.} At what time does the ball reach the second player if the second player is standing about 17.5 feet away?

\textbf{c.} If the second player heads the ball at an angle of 75°, an initial speed of 8 feet per second, and at a height of 4.75 feet, approximately how long does the ball stay in the air from the time it is first kicked until it lands?

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{soccer_graph.png}
\caption{Graph of soccer ball path}
\end{figure}

\textit{SOLUTION:}

\textbf{a.} The horizontal position of the ball can be modeled by \( x = tv_0 \cos \theta \). Since the point (7, 5.77) is provided at \( t = 0.4 \),

\textbf{b.} The vertical position of the ball can be modeled by \( y = -\frac{1}{2}gt^2 + v_0 \sin \theta \cdot t + h_0 \). With the given values, we can solve for the initial velocity and the time it takes to reach the second player.

\textbf{c.} The time the ball stays in the air can be calculated using the vertical motion equation, taking into account the new angle and initial conditions.
7-5 Parametric Equations

solve for $v_0$ using $x = 7, t = 0.4$, and $\theta = 50^\circ$.

\[
\begin{align*}
x &= v_0 \cos \theta \\
7 &= 0.4 \cdot v_0 \cdot \cos 50 \\
\frac{7}{0.4 \cos 50} &= v_0 \\
27.2 &= v_0
\end{align*}
\]

The initial velocity of the ball is 27.2 feet per second.

b. To find the time the ball reaches the second player, solve for $t$ using the horizontal position function.

\[
\begin{align*}
x &= v_0 \cos \theta \\
17.5 &= t(27.2) \cdot \cos 50 \\
\frac{17.5}{27.2 \cos 50} &= v_0 \\
1.00 &= v_0
\end{align*}
\]

The ball will reach the second player in about 1 second.

c. To find the time that the ball is in the air after the second player hits it, solve for $t$ when the vertical position function equals 0. Since velocity is given in feet, use 32 feet per second squared for $g$.

\[
\begin{align*}
y &= tv_0 \sin \theta - \frac{1}{2} gt^2 + h_0 \\
0 &= t(8) \sin 75 - \frac{1}{2} (32)t^2 + 4.75 \\
7.73 &= 16t^2 + 4.75
\end{align*}
\]

Graph the equation for the vertical position. Use 2: zero function on the CALC menu to find the time $t$ for when the ball will hit the ground. The value is about 0.84 second.

The ball was in the air for 1 second before it was hit by the second player, and then stayed in the air for an additional 0.84 second. In total, the ball was in the air for about 1.84 seconds.
7-5 Parametric Equations

52. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate a cycloid, the curve created by the path of a point on a circle with a radius of 1 unit as it is rolled along the x-axis.
   
a. **GRAPHICAL** Use a graphing calculator to graph the parametric equations \( x = t - \sin t \) and \( y = 1 - \cos t \), where \( t \) is measured in radians.
   
b. **ANALYTICAL** What is the distance between \( x \)-intercepts? Describe what the \( x \)-intercepts and the distance between them represent.
   
c. **ANALYTICAL** What is the maximum value of \( y \)? Describe what this value represents and how it would change for circles of differing radii.

**SOLUTION:**

a. 

![Graph of cycloid](image)

b. The \( x \)-intercepts occur when the vertical position function equals 0.

\[
\begin{align*}
y &= 1 - \cos t \\
0 &= 1 - \cos t \\
-1 &= -\cos t \\
1 &= \cos t
\end{align*}
\]

The solution to this equation occur when \( t \) equals integer multiples of \( 2\pi \). Therefore, the distance between any two consecutive \( x \)-intercepts is \( 2\pi \). The \( x \)-intercepts represent the instances when the point on the circle touches the \( x \)-axis as it is rolled. Since the entire circumference of the circle will touch the \( x \)-axis as it is rolled, the distance between the \( x \)-intercepts will be equal to the circumference of the circle, which is equal to \( 2\pi \).

\[c. \text{ The maximum value of } y \text{ is } 2. \text{ This value represents the maximum height the point reaches as the circle is rolled along the } x \text{-axis. It is equal to the diameter of the circle. A circle with a radius of } r \text{ will produce a maximum } y \text{-value of } 2r.\]
53. **CHALLENGE** Consider a line $l$ with parametric equations $x = 2 + 3t$ and $y = -t + 5$. Write a set of parametric equations for the line $m$ perpendicular to $l$ containing the point (4, 10).

**SOLUTION:**
Write the parametric equations in rectangular form to find the slope of $l$.

Solve for $t$.

\[
\begin{align*}
y &= -t + 5 \\
y - 5 &= -t \\
5 - y &= t
\end{align*}
\]

Substitute for $t$.

\[
\begin{align*}
x &= 2 + 3t \\
x &= 2 + 3(5 - y) \\
x &= 2 + 15 - 3y \\
x &= 17 - 3y \\
x - 17 &= -3y \\
\frac{x - 17}{3} &= y \\
\frac{-1}{3}x + \frac{17}{3} &= y
\end{align*}
\]

Since the slope of $l$ is $-\frac{1}{3}$, the slope of $m$ is 3.

Make a table of values. Let $t = 0$ represent the first point (4, 10) and let $t$ be in intervals of 1.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>16</td>
</tr>
</tbody>
</table>

Since the slope of the line is 3, the $y$–values will increase by 3 units each time $x$ increases by 1 unit. Find an equation for $x$ in terms of $t$. Since $x$ increases by increments of 1, $m = 1$.

\[
\begin{align*}
x &= mt + b \\
n &= (1)(0) + b \\
n &= b
\end{align*}
\]

An equation for $x$ is $x = t + 4$. Repeat the process to find an equation for $y$ in terms of $t$. Since $y$ increases by increments of 3, $m = 3$.

\[
\begin{align*}
y &= mt + b \\
n &= 3(0) + b \\
n &= b
\end{align*}
\]

An equation for $y$ is $y = 3t + 10$. 
7-5 Parametric Equations

54. Writing in Math  Explain why there are infinitely many sets of parametric equations to describe one line in the $xy$–plane.

**SOLUTION:**
Parametric equations are written using a point on the line and a parallel vector. An infinite number of equations can be written using an infinite number of points on any line.

55. REASONING  Determine whether parametric equations for projectile motion can apply to objects thrown at an angle of $90^\circ$. Explain your reasoning.

**SOLUTION:**
The horizontal distance is modeled by the cosine function, which is 0 at $90^\circ$. This would imply that the projectile has no horizontal movement. The corresponding parametric equation would be $x = 0$.

56. CHALLENGE  A line in three–dimensional space contains the points $P(2, 3, -8)$ and $Q(-1, 5, -4)$. Find two sets of parametric equations for the line.

**SOLUTION:**

Make a table of values. Let $t = 0$ represent $P$ and $t = 1$ represent $Q$.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>3</td>
<td>-8</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>5</td>
<td>-4</td>
</tr>
</tbody>
</table>

Find an equation for $x$ in terms of $t$. Since $x$ decreases from 2 to $-1$, $m = -3$.

\[
x = mt + b
\]

\[
2 = (-3)(0) + b
\]

\[
2 = b
\]

An equation for $x$ is $x = -3t + 2$. Repeat the process to find an equation for $y$ in terms of $t$. Since $y$ increases from 3 to 5, $m = 2$.

\[
y = mt + b
\]

\[
3 = 2(0) + b
\]

\[
3 = b
\]

An equation for $y$ is $y = 2t + 3$. Repeat the process to find an equation for $z$ in terms of $t$. Since $z$ increases from $-8$ to $-4$, $m = 4$.

\[
z = mt + b
\]

\[
-8 = 4(0) + b
\]

\[
-8 = b
\]

An equation for $z$ is $z = 4t - 8$.

A second set of parametric equations for the line can be found if we let $t = 0$ for $Q$ and repeat the same process: $x = -3t - 1$, $y = 2t + 5$, $z = 4t - 4$.

57. Writing in Math  Explain the advantage of using parametric equations versus rectangular equations when analyzing the horizontal and vertical components of a graph.

**SOLUTION:**
Parametric equations show both horizontal and vertical positions of an object over time, while rectangular equations can only show one or the other.
Graph each equation at the indicated angle.

58. \( \frac{(x')^2}{9} - \frac{(y')^2}{4} = 1 \) at a 60° rotation from the xy-axis

**SOLUTION:**

\[ \frac{(x')^2}{9} - \frac{(y')^2}{4} = 1 \] at a 60°

Find the equations for \( x \) and \( y \).

\[ x' = x \cos \theta + y \sin \theta \]
\[ x' = \frac{1}{2} x + \frac{\sqrt{3}}{2} y \]
\[ y' = y \cos \theta - x \sin \theta \]
\[ y' = \frac{1}{2} y - \frac{\sqrt{3}}{2} x \]

Substitute these values into the original equation.

\[ \frac{(x')^2}{9} - \frac{(y')^2}{4} = 1 \]
\[ 4(x')^2 - 9(y')^2 = 36 \]
\[ 4 \left( \frac{1}{2} x + \frac{\sqrt{3}}{2} y \right)^2 - 9 \left( \frac{1}{2} y - \frac{\sqrt{3}}{2} x \right)^2 = 36 \]
\[ 4 \left( \frac{1}{4} x^2 + \frac{\sqrt{3}}{2} xy + \frac{3}{4} y^2 \right) - 9 \left( \frac{1}{4} y^2 - \frac{\sqrt{3}}{2} xy + \frac{3}{4} x^2 \right) = 36 \]
\[ x^2 + 2\sqrt{3} xy + 3y^2 - \frac{9}{4} x + \frac{9\sqrt{3}}{2} xy - \frac{27}{4} y^2 = 36 \]
\[ 4x^2 + 8\sqrt{3} xy + 12y^2 - 9x + 18\sqrt{3} xy - 27y^2 = 144 \]
\[ -23x^2 + 26\sqrt{3} xy + 3y^2 - 144 = 0 \]

Graph the equation by solving for \( y \).

\[ 3y^2 + (26\sqrt{3} x)y - 23x^2 - 144 = 0 \]

Use the quadratic formula.

\[ y = \frac{-26\sqrt{3} x \pm \sqrt{(26\sqrt{3} x)^2 - 4(3)(-23x^2 - 144)}}{2(3)} \]
\[ y = \frac{-26\sqrt{3} x \pm \sqrt{2028x^2 + 276x^2 + 1728}}{6} \]
\[ y = \frac{-26\sqrt{3} x \pm \sqrt{2304x^2 + 1728}}{6} \]

Graph the conic using your graphing calculator.
59. \((x')^2 - (y')^2 = 1\) at a 45° rotation from the \(xy\)-axis

**Solution:**

Find the equations for \(x\) and \(y\).

\[ x' = x \cos \theta + y \sin \theta \]
\[ y' = y \cos \theta - x \sin \theta \]

Substitute these values into the original equation.

\[(x')^2 - (y')^2 = 1\]
\[\left(\frac{\sqrt{2}}{2} x + \frac{\sqrt{2}}{2} y\right)^2 - \left(\frac{\sqrt{2}}{2} y - \frac{\sqrt{2}}{2} x\right)^2 = 1\]
\[\frac{1}{2} x^2 + xy + \frac{1}{2} y^2 - \frac{1}{2} y^2 - xy + \frac{1}{2} x^2 = 1\]
\[\frac{1}{2} x^2 + xy - \frac{1}{2} y^2 + xy - \frac{1}{2} x^2 = 1\]
\[2xy = 1\]

Graph the conic using your graphing calculator.
7-5 Parametric Equations

Write an equation for the hyperbola with the given characteristics.

60. vertices (5, 4), (5, −8); conjugate axis length of 4

Solution:

Because the \( x \)-coordinates of the vertices are the same, the transverse axis is vertical, and the standard form of the equation is

\[
\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1.
\]

The center is located at the midpoint of the vertices, or (5, −2). So, \( h = 5 \) and \( k = -2 \). Because the vertices are 12 units apart, \( 2a = 12 \), \( a = 6 \), and \( a^2 = 36 \). The length of the conjugate axis is 4 units, so \( 2b = 4 \), \( b = 2 \), and \( b^2 = 4 \).

Using the values of \( h, k, a, \) and \( b \), the equation for the hyperbola is

\[
\frac{(y + 2)^2}{36} - \frac{(x - 5)^2}{4} = 1.
\]

61. transverse axis length of 4; foci (3, 5), (3, −1)

Solution:

Because the \( x \)-coordinates of the vertices are the same, the transverse axis is vertical, and the standard form of the equation is

\[
\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1.
\]

The center is located at the midpoint of the foci, or (3, 2). So, \( h = 3 \) and \( k = 2 \). The transverse axis has a length of \( 2a \) units. So, \( 4 = 2a \), \( a = 2 \), and \( a^2 = 4 \). Because the foci are 6 units apart, \( 2c = 6 \), \( c = 3 \), and \( c^2 = 9 \). Since \( c^2 = a^2 + b^2 \), \( b^2 = 5 \). Using the values of \( h, k, a, \) and \( b \), the equation for the hyperbola is

\[
\frac{(y - 2)^2}{4} - \frac{(x - 3)^2}{5} = 1.
\]

62. White House

There is an open area south of the White House known as The Ellipse. Write an equation to model The Ellipse. Assume that the origin is at its center.

Solution:

Because the major axis is vertical, the standard form of the ellipse is

\[
\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1.
\]

The center is at (0, 0). So, \( h = 0 \) and \( k = 0 \). The major axis is \( 2a \) or 1057 feet. So, \( 2a = 1057 \), \( a = 528.5 \), and \( a^2 = 279,312.25 \). The minor axis is \( 2b \) or 880 feet. So, \( 2b = 880 \), \( b = 440 \), and \( b^2 = 193,600 \). Using the values of \( h, k, a, \) and \( b \), the equation for the ellipse is

\[
\frac{x^2}{193600} + \frac{y^2}{279312.25} = 1.
\]
7-5 Parametric Equations

Simplify each expression.

63. \(\frac{\sin x}{\csc x - 1} + \frac{\sin x}{\csc x + 1}\)

**SOLUTION:**

\[
\frac{\sin x}{\csc x - 1} + \frac{\sin x}{\csc x + 1} = \frac{\sin x(\csc x + 1)}{\sin x(\csc x - 1)} + \frac{\sin x(\csc x - 1)}{\sin x(\csc x + 1)}
\]

\[
= \frac{\csc^2 x - 1}{\sin x} + \frac{\csc^2 x - 1}{\sin x}
\]

\[
= \csc^2 x - \frac{\sin x}{\sin x} - \sin x
\]

\[
= 2\csc^2 x
\]

= \(2\tan^2 x\)

64. \(\frac{1}{1 - \cos x} + \frac{1}{1 + \cos x}\)

**SOLUTION:**

\[
\frac{1}{1 - \cos x} + \frac{1}{1 + \cos x} = \frac{1}{1 + \cos x} + \frac{1}{1 - \cos x}
\]

\[
= \frac{1 - \cos^2 x}{1 + \cos x + 1 - \cos x}
\]

\[
= \frac{\sin^2 x}{2}
\]

= \(2\csc^2 x\)

Use the properties of logarithms to rewrite each logarithm below in the form \(a \ln 2 + b \ln 3\), where \(a\) and \(b\) are constants. Then approximate the value of each logarithm given that \(\ln 2 \approx 0.69\) and \(\ln 3 \approx 1.10\).

65. \(\ln 54\)

**SOLUTION:**

\[
\ln 54 = \ln (2 \cdot 3^3)
\]

\[
= \ln 2 + \ln 3^3
\]

\[
= \ln 2 + 3 \ln 3
\]

Use substitution to evaluate.

\[
\ln 2 + 3 \ln 3 \approx (0.69) + 3(1.10)
\]

\[
\approx 3.99
\]
7-5 Parametric Equations

66. \( \ln 24 \)

**SOLUTION:**

\[
\ln 24 = \ln (2^3 \cdot 3) \\
= \ln 2^3 + \ln 3 \\
= 3 \ln 2 + \ln 3
\]

Use substitution to evaluate.

\[
3 \ln 2 + \ln 3 \approx 3(0.69) + (1.10) \\
\approx 3.17
\]

67. \( \ln \frac{8}{3} \)

**SOLUTION:**

\[
\ln \frac{8}{3} = \ln 8 - \ln 3 \\
= \ln 2^3 - \ln 3 \\
= 3 \ln 2 - \ln 3
\]

Use substitution to evaluate.

\[
3 \ln 2 - \ln 3 \approx 3(0.69) - (1.10) \\
\approx 0.97
\]

68. \( \ln \frac{9}{16} \)

**SOLUTION:**

\[
\ln \frac{9}{16} = \ln 9 - \ln 16 \\
= \ln 3^2 - \ln 2^4 \\
= 2 \ln 3 - 4 \ln 2
\]

Use substitution to evaluate.

\[
2 \ln 3 - 4 \ln 2 \approx 2(1.10) - 4(0.69) \\
\approx -0.56
\]
7-5 Parametric Equations

For each function, determine any asymptotes and intercepts. Then graph the function and state its domain.

69. \( h(x) = \frac{x}{x + 6} \)

**SOLUTION:**

The function is undefined when the denominator is equal to zero, so the domain of \( h = \{ x \mid x \neq -6, x \in \mathbb{R} \} \).

There is a vertical asymptote at the real zero of the denominator \( x = -6 \).

There is a horizontal asymptote at \( y = \frac{1}{1} \) or \( y = 1 \), the ratio of the leading coefficients of the numerator and denominator, because the degrees of the polynomials are equal.

The \( x \)-intercept is \( (0, 0) \), because 0 is the zero of the numerator. Substitute 0 for \( x \) to find the \( y \)-intercept.

\[
\begin{align*}
 h(x) &= \frac{x}{x + 6} \\
 h(0) &= \frac{0}{0 + 6} \\
 h(0) &= 0
\end{align*}
\]

They--intercept is \( (0, 0) \), because \( h(0) = 0 \).

Use a table of values to graph \( h \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-9</td>
<td>3</td>
</tr>
<tr>
<td>-8</td>
<td>4</td>
</tr>
<tr>
<td>-7</td>
<td>7</td>
</tr>
<tr>
<td>-6</td>
<td>undefined</td>
</tr>
<tr>
<td>-5</td>
<td>-5</td>
</tr>
<tr>
<td>-4</td>
<td>-2</td>
</tr>
<tr>
<td>-3</td>
<td>-1</td>
</tr>
</tbody>
</table>

70. \( h(x) = \frac{x^2 + 6x + 8}{x^2 - 7x - 8} \)

**SOLUTION:**

Factor the numerator and the denominator.

\[
\frac{x^2 + 6x + 8}{x^2 - 7x - 8} = \frac{(x + 4)(x + 2)}{(x - 8)(x + 1)}
\]
The function is undefined when the denominator is equal to zero, so the domain of \( h = \{ x \mid x \neq -1, 8, x \in \mathbb{R} \} \).

There are vertical asymptotes at the real zeros of the denominator \( x = -1 \) and \( x = 8 \).
There is a horizontal asymptote at \( y = \frac{1}{1} \) or \( y = 1 \), the ratio of the leading coefficients of the numerator and denominator, because the degrees of the polynomials are equal.

The \( x \)-intercepts are \((-4, 0)\) and \((-2, 0)\), the zeros of the numerator.

Substitute 0 for \( x \) to find the \( y \)-intercept.
\[
\hat{h}(x) = \frac{(x + 4)(x + 2)}{(x - 8)(x + 1)}
\]
\[
\hat{h}(0) = \frac{(0 + 4)(0 + 2)}{(0 - 8)(0 + 1)}
\]
\[
\hat{h}(0) = \frac{8}{-8} = -1
\]
The \( y \)-intercept is \((0, -1)\) because \( h(0) = -1 \).

Use a table of values to graph \( h \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>0.21</td>
</tr>
<tr>
<td>-6</td>
<td>0.11</td>
</tr>
<tr>
<td>-4</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-1.33</td>
</tr>
<tr>
<td>4</td>
<td>-2.4</td>
</tr>
<tr>
<td>6</td>
<td>-5.71</td>
</tr>
<tr>
<td>8</td>
<td>undefined</td>
</tr>
<tr>
<td>10</td>
<td>7.64</td>
</tr>
<tr>
<td>12</td>
<td>4.31</td>
</tr>
</tbody>
</table>

\[
71. f(x) = \frac{x^2 + 8x}{x + 5}
\]

**SOLUTION:**
Factor the numerator and the denominator.
7-5 Parametric Equations

\[
\frac{x^2 + 8x}{x + 5} = \frac{x(x + 8)}{x + 5}
\]

The function is undefined when the denominator is equal to zero, so the domain of \( f = \{ x \mid x \neq -5, x \in \mathbb{R} \} \).

There is a vertical asymptote at the real zero of the denominator \( x = -5 \). The degree of the numerator is greater than the degree of the denominator, so there is no horizontal asymptote. Because the degree of the numerator is exactly one more than the degree of the denominator, \( f \) has a slant asymptote. Using polynomial division, you can write the following.

\[
f(x) = \frac{x^2 + 8x}{x + 5} = x + 3 - \frac{15}{x + 5}
\]

Therefore, the equation of the slant asymptote is \( y = x + 3 \).

The \( x \)-intercepts are 0 and -8, the zeros of the numerator.

Substitute 0 for \( x \) to find the \( y \)-intercept.

\[
f(0) = \frac{0(0 + 8)}{0 + 5} = \frac{0}{5} = 0
\]

The \( y \)-intercept is 0 because \( f(0) = 0 \).

Use a table of values to graph \( f \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>0</td>
</tr>
<tr>
<td>-7</td>
<td>3.5</td>
</tr>
<tr>
<td>-6</td>
<td>12</td>
</tr>
<tr>
<td>-5</td>
<td>undefined</td>
</tr>
<tr>
<td>-4</td>
<td>-16</td>
</tr>
<tr>
<td>-3</td>
<td>-7.5</td>
</tr>
<tr>
<td>-2</td>
<td>-4</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{Graph of } f(x) & \\text{versus } x.
\end{align*}
\]
7-5 Parametric Equations

72. \( f(x) = \frac{x^2 + 4x + 3}{x^3 + x^2 - 6x} \)

**SOLUTION:**

Factor the numerator and the denominator.

\[
\frac{x^2 + 4x + 3}{x^3 + x^2 - 6x} = \frac{(x + 3)(x + 1)}{x(x + 3)(x - 2)} = \frac{x + 1}{x(x - 2)}
\]

The function is undefined when the denominator is equal to zero, so the domain of \( h = \{ x \mid x \neq 0, 2, x \in \mathbb{R} \} \). There are vertical asymptotes at the real zeros of the denominator \( x = 0 \) and \( x = 2 \). There is a horizontal asymptote at \( y = 0 \), because the degree of the denominator is greater than the degree of the numerator.

The \( x \)-intercept is \(-1\), the zero of the numerator.

There is no \( y \)-intercept because \( f(0) \) is undefined.

There is a removable discontinuity at \( x = -3 \).

Use a table of values to graph \( f \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-0.13</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>-3</td>
<td>undefined</td>
<td>2</td>
<td>undefined</td>
</tr>
<tr>
<td>-2</td>
<td>-0.13</td>
<td>3</td>
<td>1.33</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>4</td>
<td>0.63</td>
</tr>
<tr>
<td>0</td>
<td>undefined</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Solve each equation**

73. \( \sqrt{3z - 5} - 3 = 1 \)

**SOLUTION:**

\[
\sqrt{3z - 5} - 3 = 1
\]

\[
\sqrt{3z - 5} = 4
\]

\[
3z - 5 = 16
\]

\[
z = 7
\]
7-5 Parametric Equations

74. \( \sqrt{5n-1} = 0 \)

\[ \begin{align*}
\sqrt{5n-1} & = 0 \\
5n - 1 & = 0 \\
5n & = 1 \\
n & = \frac{1}{5}
\end{align*} \]

75. \( \sqrt{2c+3} - 7 = 0 \)

\[ \begin{align*}
\sqrt{2c+3} - 7 & = 0 \\
\sqrt{2c+3} & = 7 \\
2c + 3 & = 49 \\
2c & = 46 \\
c & = 23
\end{align*} \]

76. \( \sqrt{4a} + 8 + 8 = 5 \)

\[ \begin{align*}
\sqrt{4a} + 8 & = 5 \\
\sqrt{4a} & = -3
\end{align*} \]

Since the square root of a real number cannot be a negative number, there is no solution to this equation.
7-5 Parametric Equations

77. SAT/ACT  With the exception of the shaded squares, every square in the figure contains the sum of the number in the square directly above it and the number in the square directly to its left. For example, the number 4 in the unshaded square is the sum of the 2 in the square above it and the 2 in the square directly to its left. What is the value of x?

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A 7  
B 8  
C 15  
D 23  
E 30

**SOLUTION:**

Continue to fill in the squares following the pattern until the square with the x is found.

The correct answer is E.
7-5 Parametric Equations

78. Jack and Graham are performing a physics experiment in which they will launch a model rocket. The rocket is supposed to release a parachute 300 feet in the air, 7 seconds after liftoff. They are firing the rocket at a 78° angle from the horizontal. To protect other students from the falling rockets, the teacher needs to place warning signs 50 yards from where the parachute is released. How far should the signs be from the point where the rockets are launched?

F 122 yards
G 127 yards
H 133 yards
J 138 yards

SOLUTION:
Sketch a diagram of the launch.

To find the distance of the farthest sign from the point of the launch, the horizontal position \( x \) of the rocket at the time when the parachute is released must be found. Use the vertical position function to find the initial velocity \( v_0 \) of the rocket.

\[
y = tv_0 \sin \theta - \frac{1}{2} gt^2 + h_0
\]

\[
300 = 7 \cdot v_0 \cdot \sin 78 - \frac{1}{2} (32)(7)^2 + 0
\]

\[
300 = 6.847v_0 - 784
\]

\[
1084 = 6.847v_0
\]

\[
158.3 = v_0
\]

Find the horizontal position of the rocket when the parachute is released.

\[
x = tv_0 \cos \theta
\]

\[
= (7)(158.3) \cos 78
\]

\[
\approx 230.39
\]

The rocket releases its parachute about 230 feet from the launch point. This is approximately 76.67 yards. Adding an additional 50 yards to this distance places the farthest sign 126.67 yards from the launch point. The correct answer is G.

79. FREE RESPONSE   An object moves along a curve according to \( y = \frac{10\sqrt{3t} + \sqrt{496 - 2304t}}{62} \), \( x = \sqrt{t} \).

a. Convert the parametric equations to rectangular form.
b. Identify the conic section represented by the curve.
c. Write an equation for the curve in the \( x'y' \)-plane, assuming it was rotated 30°.
d. Determine the eccentricity of the conic.
e. Identify the location of the foci in the \( x'y' \)-plane, if they exist.
7-5 Parametric Equations

**SOLUTION:**

a. Solve for $t$.

$$x = \sqrt{t}$$

$$x^2 = t.$$  

Substitute for $t$.

$$y = \frac{10\sqrt{3x} \pm \sqrt{496 - 2304t}}{62}$$

$$y = \frac{10\sqrt{3x^2} \pm \sqrt{496 - 2304(x^2)}}{62}$$

$$62y = 10\sqrt{3x} \pm \sqrt{496 - 2304x^2}$$

$$62y - 10\sqrt{3}x = \sqrt{496 - 2304x^2}$$

$$(62y - 10\sqrt{3}x)^2 = 496 - 2304x^2$$

$$3844y^2 - 1240\sqrt{3}xy + 300x^2 = 496 - 2304x^2$$

$$3844y^2 - 1240\sqrt{3}xy + 2604x^2 = 496$$

$$3ly^2 - 10\sqrt{3}xy + 21x^2 = 4$$

b. Use a graphing calculator to graph the equation. The equation needs to be solved for $y$ in order to be graphed.

Use the equation

$$y = \frac{10\sqrt{3x^2} \pm \sqrt{496 - 2304(x^2)}}{62}.$$  

![Graph of the equation](image)

The conic section is an ellipse.

c. Find the equations for $x$ and $y$.

$$x = x' \cos \theta - y' \sin \theta$$

$$x = x' \cos 30^\circ - y' \sin 30^\circ$$

$$x = \frac{\sqrt{3}}{2} x' - \frac{1}{2} y'$$

$$y = x' \sin \theta + y' \cos \theta$$

$$y = x' \sin 30^\circ + y' \cos 30^\circ$$

$$y = \frac{1}{2} x' + \frac{\sqrt{3}}{2} y'$$

Substitute into the original equation.
7-5 Parametric Equations

\[
3y^2 - 10\sqrt{3}xy + 21x^2 = 4
\]

\[
31\left(\frac{x' - \sqrt{3}y'}{2}\right)^2 + 10\sqrt{3}\left(\frac{\sqrt{3}x' - y'}{2}\right)\left(\frac{\sqrt{3}x' - y'}{2}\right) + 21\left(\frac{\sqrt{3}x' - y'}{2}\right)^2 = 4
\]

\[
31\left(\frac{(x')^2 + 2\sqrt{3}x'y' + 3(y')^2}{4}\right) - 10\sqrt{3}\left(\frac{\sqrt{3}(x')^2 - x'y' + 3x'y' + \sqrt{3}(y')^2}{4}\right) + 21\left(\frac{3(x')^2 - 2\sqrt{3}x'y' + (y')^2}{4}\right) = 4
\]

\[
\frac{21(x')^2 + 62\sqrt{3}x'y' + 9(y')^2}{4} + \frac{-30(x')^2 + 10\sqrt{3}x'y' - 30\sqrt{3}x'y' - 30(y')^2}{4} + \frac{63(x')^2 - 42\sqrt{3}x'y' + 21(y')^2}{4} = 4
\]

\[
\frac{64(x')^2 + 144(y')^2}{4} = 4
\]

\[
16(x')^2 + 36(y')^2 = 4
\]

\[
4(x')^2 + 9(y')^2 = 1
\]

d. The equation \(4(x')^2 + 9(y')^2 = 1\) can be written as \(\frac{x'}{1} + \frac{y'}{1} = 1\). So, \(a^2 = \frac{1}{4}\) and \(b^2 = \frac{1}{9}\). Use \(c^2 = a^2 - b^2\) to solve for \(c^2\).

\[
c^2 = a^2 - b^2
\]

\[
c^2 = \frac{1}{4} - \frac{1}{9}
\]

\[
c^2 = \frac{5}{36}
\]

If \(c^2 = \frac{5}{36}\), \(c = \frac{\sqrt{5}}{6}\). The eccentricity of the ellipse is \(e = \frac{c}{a}\).

\[
e = \frac{c}{a}
\]

\[
e = \frac{\sqrt{5}}{6}
\]

\[
e = \frac{6}{1} \approx 0.745
\]

The eccentricity of the ellipse is about 0.745.

e. The foci are located at \((h \pm c, k)\). Since the center of the ellipse is at the origin, the foci are located at \(\left(\frac{\sqrt{5}}{6}, 0\right)\) and \(\left(-\frac{\sqrt{5}}{6}, 0\right)\).