1. In exercise physiology studies it is sometimes important to determine the location of a person’s center of mass. This can be done with the arrangement shown below. A light plank rests on two scales, which read $F_{g1} = 380 \text{ N}$ and $F_{g2} = 320 \text{ N}$. The scales are separated by a distance of 2.00 m.

How far from Pat’s feet is his/her center of mass?

$$\sum F_y = 0: \quad +380 \text{ N} - F_y + 320 \text{ N} = 0$$

$$F_y = 700 \text{ N}$$

Take torques about her feet:

$$\sum \tau = 0: \quad -380 \text{ N}(2.00 \text{ m}) + (700 \text{ N})x + (320 \text{ N})0 = 0$$

$$x = 1.09 \text{ m}$$
2. A hungry bear weighing 700 N walks out on a beam in an attempt to retrieve a basket of food hanging at the end of the beam. The beam is uniform, weighs 200 N, and is 6.00 m long; the basket weighs 80.0 N.

(a) Draw a free-body diagram for the beam.

(b) When the bear is at $x = 1.00$ m, find the tension in the wire and the components of the force exerted by the wall on the left end of the beam.

(c) If the wire can withstand a maximum tension of 900 N, what is the maximum distance the bear can walk before the wire breaks?

(a) See the diagram.

(b) If $x = 1.00$ m, then

$$\sum F_x = (-700 \text{ N})(1.00 \text{ m}) - (200 \text{ N})(3.00 \text{ m})$$

$$- (80.0 \text{ N})(5.00 \text{ m})$$

$$+ (T \sin 60^\circ)(6.00 \text{ m}) = 0$$

Solving for the tension gives: $T = 342$ N

From $\sum F_y = 0$, $T \cos 60^\circ = 371$ N

From $\sum F_x = 0$, $R_x = 900 \text{ N} - T \sin 60^\circ = 683$ N.

(c) If $T = 900$ N:

$$\sum F_x = (-700 \text{ N})x - (200 \text{ N})(3.00 \text{ m}) - (80.0 \text{ N})(5.00 \text{ m}) + [(900 \text{ N}) \sin 60^\circ](6.00 \text{ m}) = 0$$

Solving for $x$ gives: $x = 5.13$ m
3. Uranus (the planet, not the sphincter) has a mass about 14 times the Earth’s mass, and its radius is equal to about 3.7 Earth radii. (a) By setting up ratios with the corresponding Earth values, find the free-fall acceleration at the cloud tops of Uranus. (b) Ignoring the rotation of the planet, find the minimum escape speed from Uranus.

\[ g = \frac{GM_e}{R_e^2} \]

\[ g_u = \frac{GM_u}{R_u^2} \]

(a)

\[ \frac{g_u}{g_e} = \frac{m_u}{m_e} \times \left( \frac{R_e}{R_u} \right)^2 = 140 \left( \frac{1}{3.70} \right)^2 = 102 \]

\[ g_u = (102)(380 \text{ m/s}^2) = 100 \text{ m/s}^2 \]

(b)

\[ v_{ex,e} = \sqrt{\frac{20 \times g_e}{g_e}} \quad v_{ex,u} = \sqrt{\frac{20 \times g_u}{g_u}} = 1.95 \]

\[ \frac{v_{ex,u}}{v_{ex,e}} = \sqrt{\frac{m_u}{m_e}} = \sqrt{3.70} = 1.95 \]

For the Earth, from the text’s table of escape speeds, \( v_{ex,e} = 11.2 \text{ km/s} \)

\[ v_{ex,u} = (1.95)(11.2 \text{ km/s}) = 21.8 \text{ km/s} \]
4. Voyager the spacecraft not the mini-van surveyed the surface of Jupiter’s moon Io and photographed active volcanoes spewing liquid sulfur (like a pledge after a frat party) to heights of 70 km above the surface of this moon. Find the speed with which the liquid sulfur left the volcano. Io’s mass is $8.9 \times 10^{22}$ kg, and its radius is 1,820 km.

To approximate the height of the sulfur, set

\[ \frac{mv^2}{2} = mgh \]
\[ v = \sqrt{2gh} \]
\[ v = \sqrt{2(1.73)(70,000)} \approx 500 \text{ m/s} \]

A more precise answer is given by

\[ \frac{1}{2}mv^2 = \frac{GMm}{r} - \frac{GMm}{q} \]
\[ v = \sqrt{\frac{2GM}{r} - \frac{2GM}{q}} \]
\[ v = \sqrt{\left(6.67 \times 10^{-11}\right)\left(6.96 \times 10^{24}\right)\left(\frac{1}{1.83 \times 10^5} - \frac{1}{1.89 \times 10^7}\right)} \]
\[ v = 430 \text{ m/s} \]
5. After our Sun exhausts its nuclear fuel, its ultimate fate may be to collapse to a white dwarf state, in which it has approximately the same mass as it has now, but a radius equal to the radius of the Earth. Calculate (a) the average density of the white dwarf, (b) the free-fall acceleration, and (c) the gravitational potential energy of a 1.00-kg object at its surface.

(a) \( \rho = \frac{\frac{M}{V}}{\frac{4}{3} \pi R^3} \approx \frac{\left(1.99 \times 10^{33} \text{ kg}\right)}{\left(6.37 \times 10^6 \text{ m}\right)^3} \approx 1.04 \times 10^9 \text{ kg/m}^3 \)

(b) \( g = \frac{GM}{R^2} = \frac{\left(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2\right) \left(1.99 \times 10^{33} \text{ kg}\right)}{\left(6.37 \times 10^6 \text{ m}\right)^2} = 3.27 \times 10^6 \text{ m/s}^2 \)

(c) \( U_e = -\frac{GMm}{r} = -\frac{\left(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2\right) \left(1.99 \times 10^{33} \text{ kg}\right) \left(1.00 \text{ kg}\right)}{6.37 \times 10^6 \text{ m}} = -2.06 \times 10^{45} \text{ J} \)
6. A chunk of Mark Harmon of mass 4.00 kg is attached to a spring with a force constant of 100 N/m. It is oscillating on a horizontal frictionless surface with an amplitude of 2.00 m. A second 5.00-kg chunk is dropped vertically on top of the 4.00-kg chunk as it passes through its equilibrium point. The two chunks stick together. (a) By how much does the amplitude of the vibrating system change as a result of the collision? (b) By how much does the period change? (c) By how much does the energy change? (d) Account for the change in energy.

As it passes through equilibrium, the 44-kg object has speed

\[ v_{max} = \sqrt{\frac{k}{m}} A = \sqrt{\frac{100 \text{ N/m}}{4 \text{ kg}}} \times 2 \text{ m} = 10 \text{ m/s} \]

In the completely inelastic collision momentum of the two-object system is conserved. So the new 10-kg object starts its oscillation with speed given by

\[ 4 \text{ kg}(10 \text{ m/s}) + 6 \text{ kg} \cdot 0 = 10 \text{ kg} \cdot v_{new} \]

\[ v_{new} = 4 \text{ m/s} \]

(a) The new amplitude is given by

\[ \frac{1}{2} m v_{new}^2 = \frac{1}{2} k A^2 \]

\[ 10 \text{ kg}(4 \text{ m/s})^2 = \left(100 \text{ N/m}\right) A^2 \]

\[ A = 1.26 \text{ m} \]

Thus the amplitude has decreased by 2.00 m - 1.26 m = 0.74 m.

(b) The old period was

\[ T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{4 \text{ kg}}{100 \text{ N/m}}} = 1.26 \text{ s} \]

The new period is

\[ T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{10 \text{ kg}}{100 \text{ N/m}}} = 1.99 \text{ s} \]

The period has increased by 1.99 m - 1.26 m = 0.73 m.

(c) The old energy was

\[ \frac{1}{2} m v_{max}^2 = \frac{1}{2} (4 \text{ kg})(10 \text{ m/s})^2 = 200 \text{ J} \]

The new mechanical energy is

\[ \frac{1}{2} (10 \text{ kg})(4 \text{ m/s})^2 = 80 \text{ J} \]

The energy has decreased by 120 J.

(d) The missing mechanical energy has turned into internal energy in the completely inelastic collision.